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DECEMBER 1967

"SHELL GAME" ASPECTS OF MOBILE TERMINAL ABM SYSTEMS

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ADVANCED RESEARCH PROJECTS AGENCY

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PREFACE

This memorandum is part of a RAND study of mobile air-based ABM defenses done for the Advanced Research Projects Agency of the Department of Defense and is concerned with the advantages which accrue to the defender from denying the attacker knowledge of the defense deployment. A related memorandum published as part of the same project is RM-5480-ARPA, A Theoretical Analysis of Mobile Terminal Defenses Versus Fixed Defenses Against the ICBM by W. Lucas. Other memoranda related to costs and performance of possible system designs will be published later.

SUMMARY

The memorandum discusses the "shell game" aspects of mobile terminal ballistic missile defenses, i.e., the advantages which accrue to the defender from being able to deny the attacker knowledge of the defense deployment. A two-person zero-sum game which models the mobile defense problem is summarized, and references to earlier published solutions to the game are given. The results are interpreted in terms of the marginal exchange ratio, or number of additional warheads the attacker must procure to offset a single additional interceptor and maintain a constant level of damage. The marginal exchange ratio is shown to be quite favorable to the defender if the fraction of the target system which he wishes to protect is small. A comparison is made with the case of fixed defenses whose deployment is known to the attacker. Mobile defenses provide a two-to-one advantage if the defender wishes to save more than half his target system; the advantage increases sharply as the fraction saved decreases. If the defender wishes to save less than half his target system, in fact, he need procure only two additional interceptors <u>per target saved</u> for each additional warhead <u>per target</u> procured by the attacker.

The protection of a strategic retaliatory force through a combination of mobile defense and deceptive basing is also examined. The two measures complement each other well, as the mobile defense raises the price the attacker must pay

to destroy a single target, while deceptive basing increases the number of targets at which he must pay that price. The defender who wishes to protect a fixed number of launch vehicles may choose combinations of interceptors and false targets in such a way that the cost of the defenses is roughly proportional to the square root of the number of attacking warheads. Since the cost to the attacker is proportional to the number of warheads, attempts by the attacker to overcome the defenses by procuring additional warheads result in an increasingly favorable marginal exchange ratio (for the defender) as the size of the attack force increases.

Qualitative discussions of the applicability of the model to the preferential use of area defenses and of the effects of warhead and interceptor unreliability are included, and a solution to the game used in the analysis is given in the Appendix.

ACKNOWLEDGMENTS

I wish to express my appreciation to I. Blumenthal, T. Brown, D. Emerson, W. Lucas, and J. Rosen for their comments and suggestions during the preparation of this memorandum.

CONTENTS

"SHELL GAME" ASPECTS OF MOBILE TERMINAL ABM SYSTEMS

1. INTRODUCTION

This memorandum discusses the "shell game" aspects of mobile terminal ballistic missile defense, i.e., the advantages that accrue to the defender from being able to deny the attacker knowledge of the defense deployment. The mutually reinforcing advantages of a combination of mobile defense and deceptive basing for protection of the strategic retaliatory force are also considered.

The advantage to the defender of denying the attacker knowledge of the defense deployment is found to be significant. The damage creating potential of an attack is dependent not only on the amount of the attack payload surviving the defenses, but also on how that payload is targeted. The proper use of mobile defenses results in an attack distribution which overkills undefended and lightly defended targets significantly, and at the same time fails to destroy those targets which are more heavily defended.

The defender is assumed to possess a target system consisting of equal value targets, which the attacker wishes to destroy using perfect warheads each capable of destroying a single target with certainty. The defender may defend his target system with perfect interceptors, each capable of destroying one warhead attacking the target to which the interceptor is assigned. The interceptors are assumed to be mobile (e.g., air based) so that the interceptor

deployment may be changed in less time than the attacker's intelligence cycle time and the attacker has no knowledge of the defense deployment. A comparison is then made with the case of the interceptors fixed at the targets to which they are assigned, so that the attacker has full knowledge of the interceptor deployment. The advantage which accrues to the defender from denying the attacker knowledge of the interceptor deployment is illustrated in terms of the marginal exchange ratio, i.e., the number of additional warheads required to offset an increase in the number of interceptors and maintain a constant level of destruction.

Translation of this into a marginal exchange ratio in terms of systems cost would, of course, require that the full systems be considered. That is, defense costs include not only the interceptors but also the radars, means of mobility, etc., necessary to the defense system. These elements are not considered here as they do not enter into the analysis.

The possible use by the defender of a combination of mobile defense and deceptive basing (presentation of additional false targets to the attacker) is also considered. This combination appears to be an extremely attractive defense option, since the two measures complement each other well. Mobile defense raises the "price" to the attacker of destroying a single target, while deceptive basing increases the number of targets which must be destroyed. The use of the two measures together therefore forces the attacker to

pay a high price at a large number of targets, many of them false.

The model used to analyze the mobile defenses, referred to as the mobile defense game, is not new. The game consists of two symmetric cases, referred to as the attack dominant and defense dominant games. The defense dominant game was solved by the author in the context of defense against submarine-launched ballistic missiles in [4], and both cases were solved independently by Matheson in [2]. The game is summarized in the text of this memorandum, and a complete solution, together with a proof that optimal assignment strategies exist for both players except in certain extreme cases, is given in the Appendix. This latter question was left open in previous work. A related analysis, illustrating the advantages of mobile defense when the attacker desires to achieve a fixed confidence of a fixed level of destruction rather than to maximize expected damage, is given by Lucas in [1].

2. SUMMARY OF THE MOBILE DEFENSE GAME

The game may be described as follows: There are two players, the attacker and the defender. The defender has t targets and n interceptors with which to defend them. The attacker has m warheads with which to attack the targets. The defender assigns interceptors to targets without knowledge of the attacker's assignment and the attacker assigns warheads to targets without knowledge of the defender's assignment. Those targets to which more warheads than interceptors are assigned are destroyed. The attacker wishes to maximize (and the defender to minimize) the number of targets destroyed.

Let $\mu = m/t$ and $\nu = n/t$ denote the mean number of warheads and interceptors per target. There are two basic cases, referred to as the attack dominated game and defense dominated game. If μ and ν are integers, the game is attack dominated if $\mu > \nu$, and defense dominated otherwise; the expected number of targets destroyed is greater than or less than half the number of targets respectively. If μ and ν are not integers, this correspondence does not hold precisely, but the attack (defense) dominated game may still be thought of as the case in which the attacker (defender) is stronger. (The game is not necessarily attack dominated if $\mu > \nu$ because of the fact that the defender "takes ties," i.e., if the same number of warheads and interceptors are assigned to a target, the target survives.)

The optimal strategies for both players are determined by ℓ, the largest number of interceptors which the defender should place at a single target at any time. If the game is attack dominated and 2μ is an integer*, then $\ell = 2\mu-1$. The optimal strategy for the attacker is to assign his warheads in such a way that the number of warheads assigned to each target appears to have been chosen randomly between 1 and ℓ. The optimal strategy for the defender is to defend as many targets as possible as though he also has chosen the number of interceptors per defended target at random between 1 and ℓ, and leave the remaining targets undefended. Assignment procedures which achieve these properties are discussed in the Appendix. The number of targets defended will depend on the number of interceptors available, and the probability of survival of an individual target will be $\nu/(2\mu-1) = \nu/\ell$. The defender will therefore save, on the average, one target with each ℓ interceptors.

If the game is defense dominated and 2ν is an integer, then $\ell = 2\nu$. The optimal strategy for the defender is to assign interceptors in such a way that the number of interceptors assigned to each target appears to have been chosen at random between 0 and ℓ. The optimal strategy for the attacker is to attack as many targets as possible in such

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*The assumption that 2μ is an integer if the game is attack dominated and 2ν is an integer if the game is defense dominated is made in this section for simplicity. A complete analysis of the game, without this assumption, is given in the Appendix.

a way that the number of warheads assigned to an attacked target appears to have been chosen at random between 1 and ℓ. The number of targets attacked will depend on the number of warheads available, and the probability of survival of an individual target will be $1-\mu/(2\nu + 1)$. The attacker will destroy, on the average, one target with each $\ell + 1$ warheads.

Consider, for example, the case $t = 1000$, $m = 3000$, for various n. If $n = 0$, the attacker is able to destroy all targets using only 1/3 of his warheads. For $0 < n \leq 2500$, the game is attack dominated with $\ell = 5$. The attacker will attack all targets, and the defender will defend only that fraction which he can defend according to the strategy outlined above, and will save (on the average) one target for each 5 interceptors procured. If $n > 2500$, however, the game becomes defense dominated, with $\ell = 2\nu$ (if 2ν is an integer), and the attacker no longer attacks all targets. In this case the attacker will be able to destroy one target with each $2\nu + 1$ warheads. The situation is illustrated in Fig. 1.

For purposes of comparison, the number of targets saved by fixed interceptors whose location is known to the attacker is also shown. The optimal strategy for the attacker in this case is to destroy as many targets as possible by attacking them with enough warheads to exhaust the defense plus one additional warhead to destroy the target, and leaving the remaining targets unattacked. The optimal

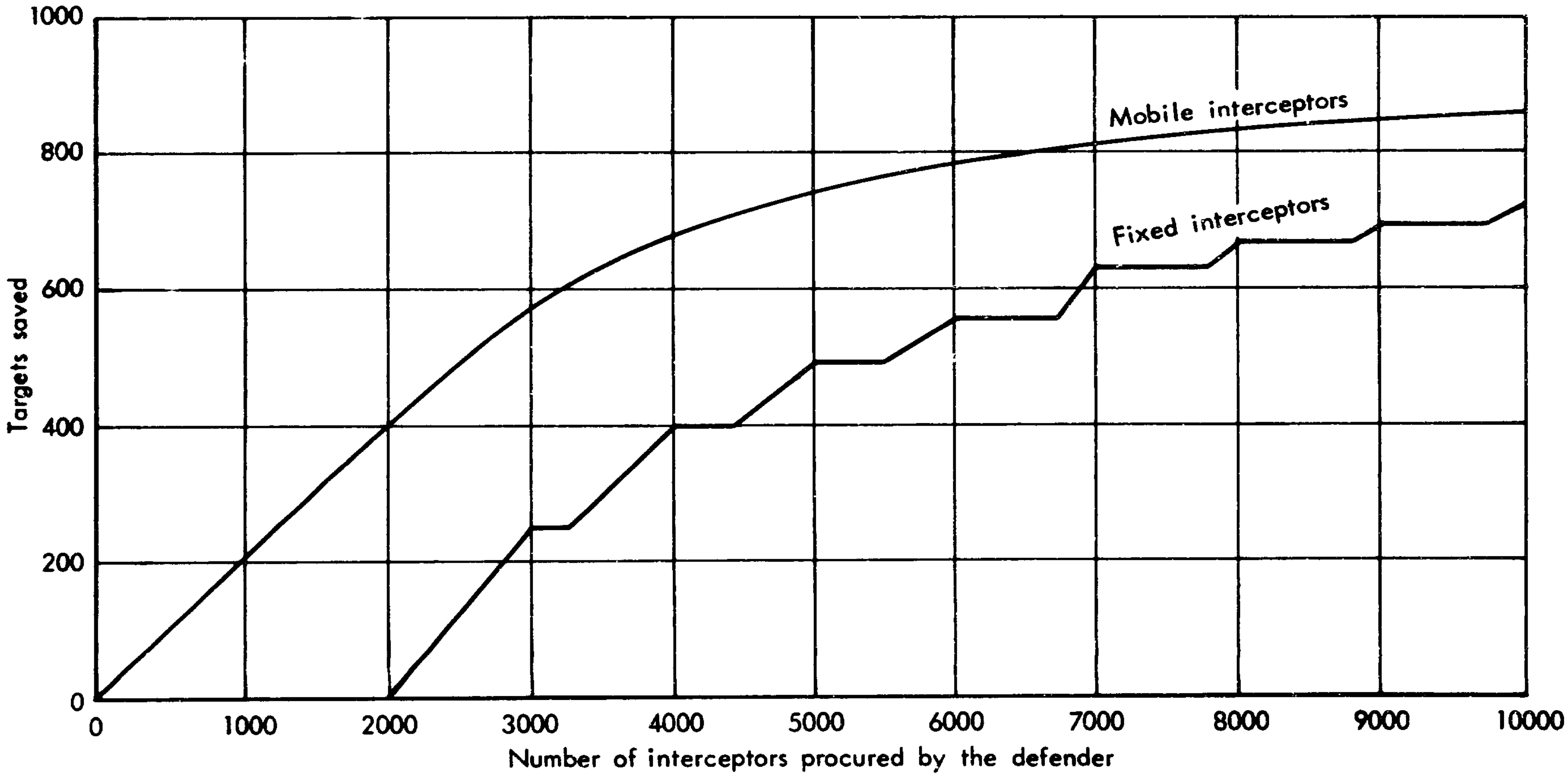

Fig. 1—Number of targets saved as a function of total number of interceptors procured by the defender. Attack of 3000 warheads against 1000 targets

strategy for the defender is to deploy his interceptors uniformly over the target system. Since the attacker has an initial superiority of 3 to 1, no targets are saved by fewer than 2000 interceptors, 1/4 of the targets by 3000 interceptors, etc. The advantage of mobile defenses is seen to be greatest for small numbers of interceptors and to decrease as the number of interceptors increases.

The step-like nature of the curve stems from the fact that additional interceptors are of no value when the attacker can avoid them by careful target selection. If the defender has between 5000 and 5500 interceptors, for example, the attacker will attack 500 targets with 6 warheads each, choosing the 500 from among those defended with 5 interceptors. Only when the number of interceptors exceeds 5500 is it necessary for him to begin attacking some targets with 7 warheads.

3. MARGINAL EXCHANGE RATIOS

One measure of the effectiveness of a defensive system is the marginal exchange ratio, the number of additional warheads the attacker must procure to offset each additional interceptor procured by the defense and yet maintain a constant level of expected damage. With this measure we can compare a defense system which denies the attacker information about the defense deployment with one which does not, and thus evaluate the relative advantage to the defender in denying deployment information by mobility or other means.

The optimal strategies for both players and the resulting level of destruction in the case of fixed defenses whose deployment is known to the attacker are described above. If ν is an integer, the attacker will destroy one target with each $\nu + 1$ warheads; destruction of d targets requires $d(\nu + 1)$ warheads. If the defender procures t additional interceptors, the attacker must procure d additional warheads to maintain the same level of damage. The marginal exchange ratio* in this case is therefore d/t.

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*Due to the requirement that integer numbers of warheads and interceptors be assigned to each target, this ratio does not in general represent the exact number of additional warheads required per additional interceptor. A single additional interceptor, for example, may require no additional warheads or may require one. The marginal exchange ratio d/t does, however, represent the approximate number of additional warheads per additional interceptor required for large increases in n, and the exact number when the defender increases n by integer multiples of t. Similar comments apply to the definition of "ratio" when interceptor deployment is unknown to the attacker.

When the attacker does not know the defense deployment, the game will be defense dominated if the expected level of damage he wishes to maintain is $d \leq t/2$, while if $t > d > t/2$, the game will be attack dominated. In the defense dominated case, if 2^{ν} is an integer, $2^{\nu} + 1$ warheads are required for each target destroyed. An increase of t interceptors therefore requires 2d additional warheads, and the marginal exchange ratio is 2d/t. In the attack dominated case, if 2^{μ} is an integer and $d < t$, each $2^{\mu} - 1$ interceptors will save a target, so that $(t - d)(2^{\mu} - 1)$ interceptors will achieve the required level of damage, and t additional warheads would be required to offset $2(t - d)$ additional interceptors, for a marginal exchange ratio of $t/2(t - d)$.

If $d = t$, the attacker must have $\mu = n$, so that t additional warheads are required to offset each additional interceptor, and the ratio is t.

The graphs of the marginal exchange ratio are shown in Fig. 2 as functions d/t, the fraction of the targets which the attacker wishes to destroy. The ratio of the two graphs provides a measure of the penalty paid by the attacker for lack of acknowledge of the defense deployment. If he wishes to destroy less than half the target system ($d/t \leq 1/2$) the penalty is a two-to-one increase in the number of warheads required, and as d approaches t this penalty increases to t-to-one.

Alternatively, the marginal exchange ratio may be viewed as the number of additional warheads which the

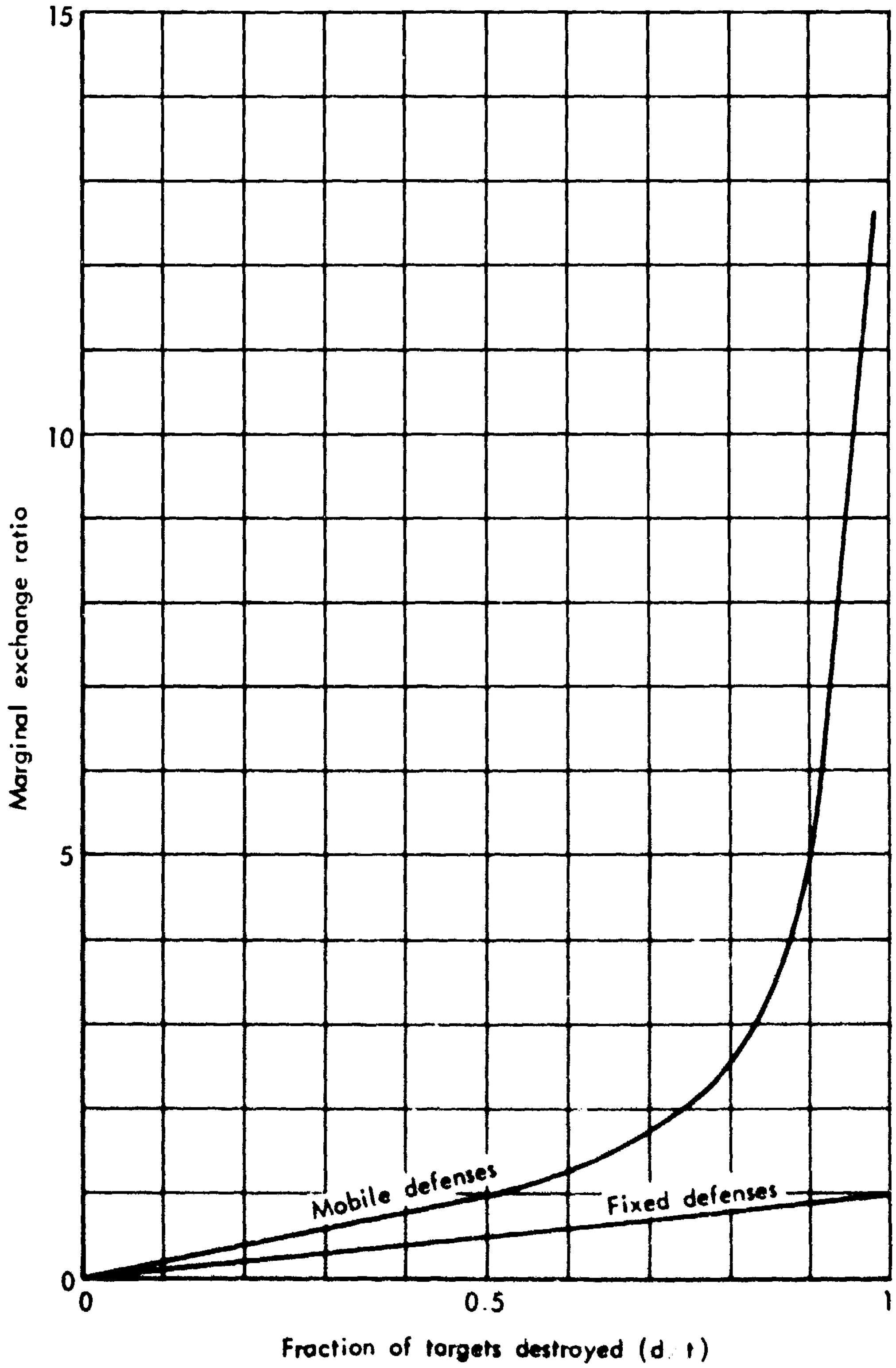

Fig. 2—Marginal exchange ratio (offense/defense) as a function of fraction of targets destroyed

attacker must procure in order to require the defender to procure a single additional interceptor if he wishes to maintain a constant level of expected survivors, s, where $s = t-d$. This level of expected survivors, s, can be thought of as the number of targets which the defender wishes to save. If $s > t/2$, the defender must match the attacker's procurement rate on a better than one-to-one basis, while for s/t near zero, the ratio becomes very favorable to the defender. Table 1 illustrates this point for a defender with a target system consisting of 1000 targets.

Table 1

MOBILE INTERCEPTOR REQUIREMENTS

TO SAVE s OUT OF 1000 TARGETS

s = Number of targets saved	10	100	500	600	750	900
Interceptors required by first 1000 warheads	10	100	500	750	1500	4500
Interceptors required by each additional 1000 warheads	20	200	1000	1250	2000	5000

4. MOBILE DEFENSE AND DECEPTIVE BASING

The marginal exchange ratio becomes increasingly favorable to the defender as the fraction of the target system which he wishes to protect decreases. If the number of targets is itself subject to the control of the defender (as is the case, for example, with retaliatory launch vehicles) he might wish to counter an increase in the size of the attack force by increasing the size of the target system rather than that of the interceptor force. Many of the targets will then be undefended and will be destroyed in the attack; those that survive will be among those which the defender chooses to defend. The undefended targets, therefore, need not be actual retaliatory launch vehicles, but only some facsimile thereof which the attacker is unable to distinguish from the real thing. The further requirement that the attacker be unable to predict the defense deployment would suggest a deceptive-basing system consisting of a number of launch sites with a lesser number of launch vehicles shifted among the sites and the defended targets chosen from among the occupied sites. The unoccupied sites would then fill the role of facsimile targets.

If deceptive basing is used, the attacker must attack the full system of launch sites (targets) while the defender need only defend the launch vehicles (occupied targets). The optimal strategies and value of the game are thus determined in a manner analogous to that without

deceptive basing by the mean number of warheads per launch site, m/t, and the mean number of interceptors per launch vehicle, n/r, where r is the number of launch vehicles. In order to insure that on the average s launch vehicles will survive an attack of m warheads, the defender may choose among a variety of combinations of r launch vehicles, t launch sites, and n interceptors which will achieve that result. If the attacker procures more warheads in an attempt to decrease the size of the surviving force, the defender may offset that increase by an appropriate increase in r, t, n, or some combination of them, and may proceed in a manner which makes the marginal exchange ratio increasingly favorable to the defender as the size of the attack force increases.

Suppose the defender wishes to choose r, t, and n in such a way that an expected number s of the launch vehicles will survive an attack of m warheads. If he chooses r such that $s < r < 2s$, he must insure the survival of over half the launch vehicles, and so must choose t and n in such a way that the game is defense dominated. Approximately $t[(2n/r)+1]/r$ warheads will be required to destroy a single launch vehicle, and the attacker is to be allowed to destroy $(r-s)$ launch vehicles. Hence r, n, and t should be chosen to satisfy the equation

$$m = \left(\frac{2n}{r} + 1\right)\left(\frac{r-s}{r}\right)t \quad \text{if } s < r < 2s,$$

which is equivalent to

$$n = \frac{r}{2}\left[\left(\frac{r}{r-s}\right)\left(\frac{m}{t}\right) - 1\right] \qquad \text{if } s < r < 2s \qquad (1)$$

If he chooses $r \geq 2s$, he may choose n and t in such a way that the game is attack dominated. Approximately $(2m/t)-1$ interceptors will be required to save a single launch vehicle; hence r, n, and t should be chosen so that

$$n = s\left(\frac{2m}{t} - 1\right) \qquad \text{if } r \geq 2s. \qquad (2)$$

Equation (1) is exact if $2n/r$ is an integer, and equation (2) is exact if $2m/t$ is an integer; otherwise they are only approximate. Equation (2) is valid for $n = 0$ only if $r = 2s$; otherwise $n = 0$ when $m/t = 1 - s/r$. Equation (2) does not contain r because the defender will only defend 2s targets; hence, so long as $m/t \geq 1$, it is irrelevant whether or not more than that are launch vehicles. The tradeoffs involved in equation (2) are illustrated in Fig. 3 for the case $s = 500$, $r = 1000$, and various values of m.

Suppose that the defender has chosen to procure at least 2s launch vehicles, for reasons other than simply insuring s survivors, and he desires to procure interceptors and additional launch sites in order to insure the desired level of survival at minimum cost, where cost is calculated in terms of interceptor equivalents. Let c be the cost

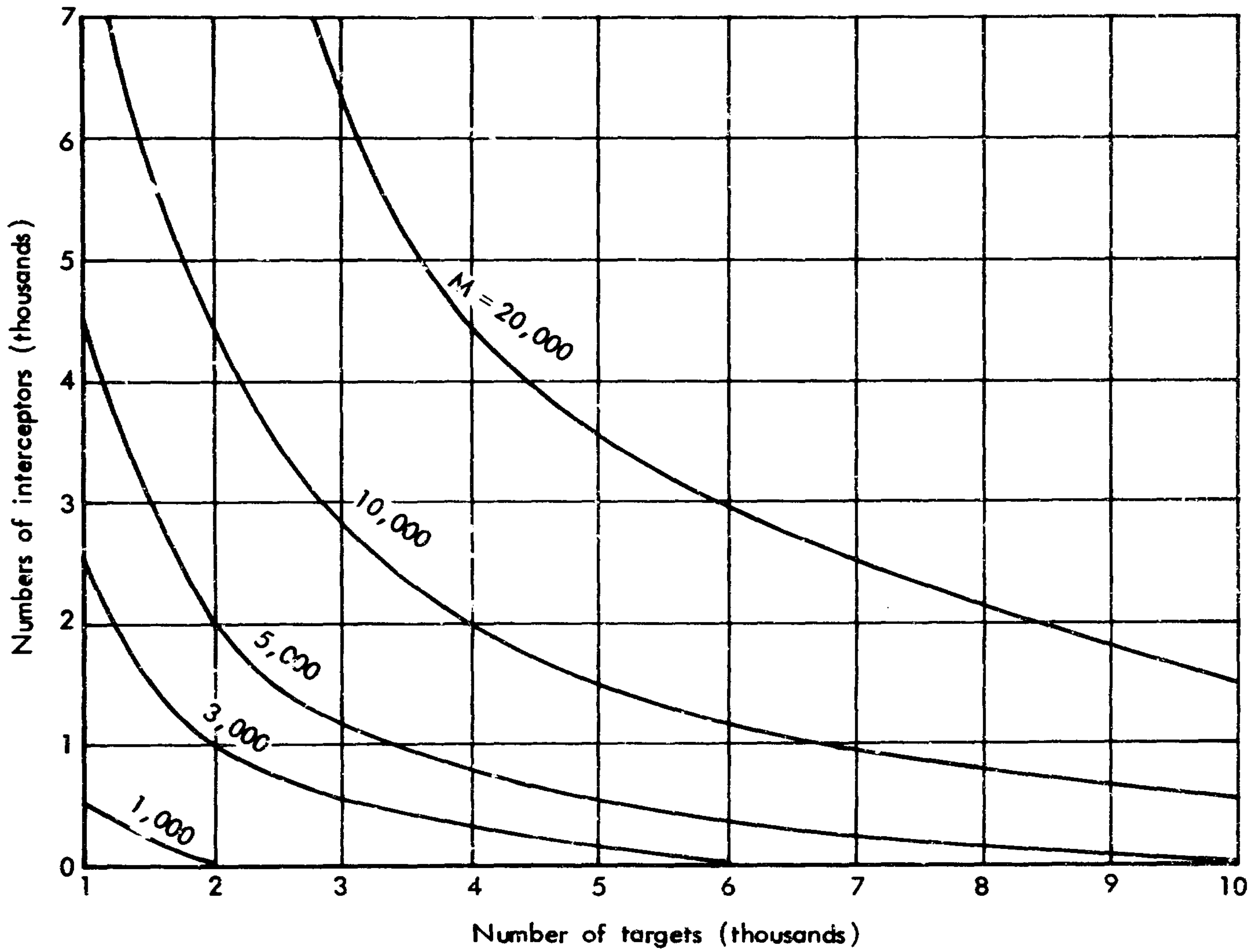

Fig. 3—Target-interceptor combinations sufficient to protect 500 of 1000 launch vehicles from an attack of m warheads

of a launch site in terms of interceptor equivalents, i.e., the ratio of launch site cost to interceptor cost. The cost of defending against an attack force of m warheads with (t − r) additional launch sites and n additional interceptors, where n is given by equation (2), is therefore

$$C_0(m, t) = c(t - r) + s(\frac{2m}{t} - 1). \qquad (3)$$

If t is treated as a continuous, rather than discrete, variable, then equation (3) is minimized by

$$t(m) = \sqrt{\frac{2ms}{c}} ,$$

which represents the approximate number of launch sites which will minimize defense costs for those values of m against which a mix of additional launch sites and interceptors should be procured.

If c is large (launch sites expensive in comparison with interceptors), then for small values of m (such that $t(m) < r + 1$), the defender should not procure any additional launch sites, but should defend only with interceptors; if c is small, then for small m (such that $t(m) > m$), the game with $t(m)$ targets and n warheads is defense dominated. In this case the defender's interceptor requirements are not given by equation (2), and a different solution must be commuted. Regardless of the value of c, however, for m sufficiently large,

$r + 1 \leq t(m) \leq m$, and the defender should use a mix of approximately $t(m)$ launch sites and $n(m)$ interceptors, where

$$n(m) = \sqrt{2cms} - s$$

is obtained by substituting $t(m)$ into equation (2).

The approximate cost to the defender of achieving the required level of expected survivors against an attack of m warheads will therefore be

$$C(m) = 2\sqrt{2cms} - cr - s \qquad (4)$$

for m sufficiently large so that $r + 1 \leq t(m) \leq m$. The marginal exchange ratio in this case is the reciprocal of the derivative of $C(m)$, or $\sqrt{m/2cs}$, and is shown in Fig. 4 as a function of m for several values of c and s. The marginal exchange ratio thus becomes increasingly large as the size of the attack force increases. It is also interesting to note that, regardless of the relative costs of interceptors and launch sites, the optimal mix for the defender against large attacks requires that approximately equal amounts be spent on interceptors and launch sites.

If the defender had not chosen to procure at least 2s launch vehicles for other reasons, some additional reduction in cost would be realizable through optimization

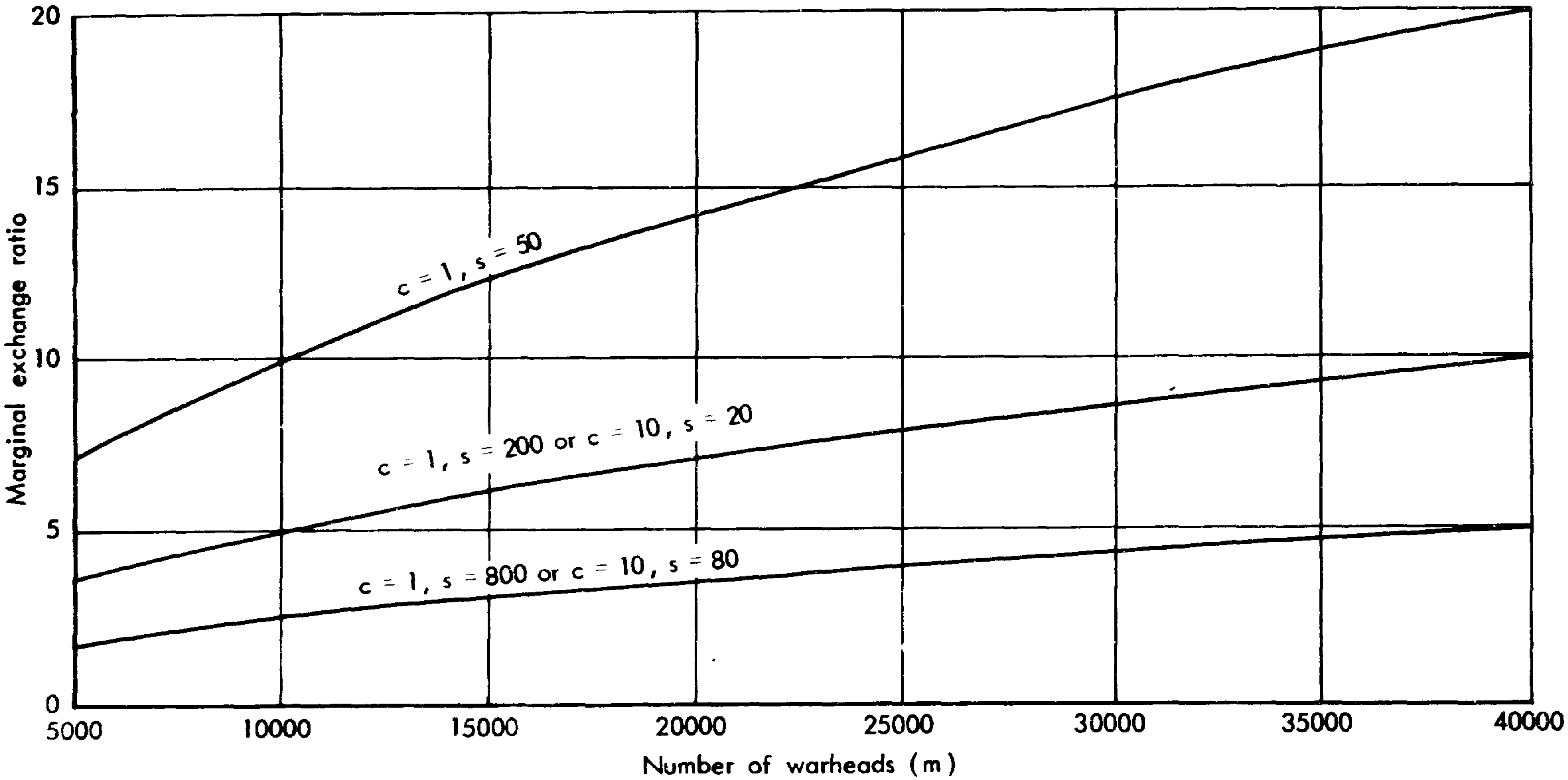

Fig. 4—Marginal exchange ratio (equivalent interceptors per warhead) when the Defender wishes to save s launch vehicles using a combination of interceptor and launch sites, with launch site cost = c equivalent interceptors

over the number of launch vehicles as well. For small values of m this reduction might be significant, but as m increases the defender would procure additional launch vehicles until he reached $r = 2s$ so the results for large m would be the same.

The conclusion to be drawn from this analysis is not that the combination of mobile defense and deceptive basing is a good means of defending a retaliatory force against a particular attack level. Whether or not this is true depends upon assumptions about the relative costs of offense and defense forces, which we have not made here, and upon the alternatives available. The analysis does show, however, that whatever the relative costs, if the defender wishes to protect a fixed number of launch vehicles against an increasing attack threat, there is a point at which it becomes more expensive for the attacker to attempt to overcome the defense than for the defender to meet that attempt, and that it becomes increasingly cheaper for the defender to counter further increases in the attack size.

5. COMPARISON WITH PREFERENTIAL AREA DEFENSE

The mobile defense model described above has much in common with the preferential use of fixed area defense, i.e., fixed ground based interceptors each capable of defending any target in the target system, with defense decisions being made as the battle progresses. However, the model seems less adequate as a description of a preferential area defense system for two reasons, relating to the problems of timing assumptions and defense of the interceptor force.

The problem of timing assumptions is the less serious, and does not greatly detract from the usefulness of the model in exhibiting the value of preferential defense. With a preferential defense system the defender need not precommit his interceptors to specific targets, but may commit them as the battle progresses, with commitment decisions being made on the basis of information available at that time. The model, which assumes precommitment, thus provides only a lower bound on the effectiveness of preferential defense.

The problem of defense of the interceptor force is perhaps more serious. William Lucas has observed* that if the preferential defense system must also provide its own defense, then much of the advantage of its preferential defense capability may be lost. If the system contains a single vital point, such as a control center or central

*Private communication.

interceptor storage facility, which must be protected if the system is to function, then the optimal strategy for the attacker is to attack that point with enough warheads to exhaust the interceptor force, then shift to the target system itself. The defender has no choice but to defend the vital point as long as it is under attack, thus exhausting his interceptor supply before the attack on the target system begins. The prime virtue of preferential defense, the ability of each interceptor to effectively neutralize more than a single warhead by forcing the attacker to overkill those targets which are undefended or underdefended, has thus been negated.

If the system contains no single vital point the problem is less serious, but the attacker can still degrade the defenses to some extent by attacking the defense system. At the other extreme, for example, suppose that each interceptor is sited separately in such a way that one warhead can destroy a single undefended interceptor, and the interceptors can defend each other. If the attacker sends a single warhead against each interceptor site, then the defender can protect at best half the interceptor force by using the other half to defend the half being protected and allowing the attacker to destroy the empty sites of the interceptors used in that defense. The attacker can therefore negate half the interceptor force at a two-for-one warhead-interceptor exchange ratio, which may be much

better than he could do by attacking the targets themselves.

The situation is quite different in the case of mobile defenses, however. If the mobile defenses are capable of self-defense, then no advantage accrues to the attacker from attempting to attack the defenses rather than the targets themselves. Because of his uncertainty concerning interceptor location, it is impossible for the attacker to exhaust the interceptor force more quickly by attacking the defenses than by attacking the targets directly.

6. THE ASSUMPTION OF PERFECT RELIABILITY

The real world, of course, contains neither perfect warheads nor perfect interceptors, and for that reason quantitative conclusions drawn from analysis of a model which assumes perfection are approximations at best. In circumstances in which qualitative assumptions hold which approximate the quantitative assumptions made in the model, however, they may be reasonable approximations, and qualitative conclusions drawn from the model should be valid.

The qualitative assumptions corresponding to the assumptions of perfect warhead and interceptor reliability are that the warheads are sufficiently reliable to achieve a high confidence of single shot kill; and that the combination of interceptor reliability and attack level is such that the primary kill mechanism at defended targets is exhaustion of the defenses rather than leakage through them. The qualitative conclusions which follow from these assumptions are that ABM defense systems which deny the attacker knowledge of their deployment can effectively negate amounts of attacking payload far out of proportion to their actual strength, especially when the defender wishes to protect only a small portion of his target system against an attacker who has significant overkill capability against the undefended target system. This occurs because the defense system raises the price of destruction significantly at those targets which the defender chooses to

defend, and the attacker, because of his lack of knowledge of the defense deployment, is forced to pay the higher price even at undefended targets. The cost of the defense system is therefore proportional to the number of targets which the defender wishes to defend, while the cost to the attacker of overcoming the defenses is proportional to the size of the total target system. As a result, the combination of mobile defense and deceptive basing, which acts to raise both the price of destroying a single target and number of targets, appears to provide an attractive method of protecting targets such as retaliatory launch vehicles.

If the single shot kill probability of an attacking warhead against a defended target is low, the advantages of mobile defense, especially in small deployments, should be even greater. With perfect warheads, a single warhead per target is sufficient to destroy undefended targets, and the attacker may target all remaining warheads in a manner which best overcomes the defenses at defended targets. All undefended targets are still destroyed, so that this targeting policy does not penalize the attacker at undefended targets. As warhead reliability decreases, however, the value of assigning multiple warheads to undefended targets increases, and changes in targeting to overcome defenses at defended targets result in a decreased probability of destruction of undefended targets.

On the other hand, as the interceptor reliability decreases, or the attack level increases, exhaustion is

gradually replaced by leakage through the defenses as the primary kill mechansim. As this happens, the value of the defenses in forcing the attacker to attempt exhaustion at large numbers of undefended targets decreases. Even if the interceptors have a reliability of 95 percent, for example, the probability that at least one warhead will penetrate the defenses at a target attacked with 15 warheads and defended with at least 15 interceptors is 55 percent. Fewer additional warheads are therefore required to offset additional interceptors, and the marginal exchange ratio decreases as the force levels increase. This is in contrast to the situation described in Section 3, in which the marginal exchange ratio was independent of force size.

At least two types of warhead reliability and two types of interceptor reliability should be considered in the analysis of battles with unreliable forces. These two types of unreliability may be referred to as launch reliability and terminal reliability, although the precise meaning of the terms is somewhat different for warheads than for interceptors. Warhead launch reliability refers to the probability that a warhead assigned to a specific target reaches the target and appears to the defender as an attacking warhead, while terminal reliability refers to the probability that such a warhead actually destroys the target if it is not destroyed by the defenses. The distinction is important because warheads that reach the target may engage an interceptor even though they would not have destroyed

the target had they not been intercepted, while warheads which fail to reach the target do not. A force of m warheads of launch reliability p and terminal reliability one is, therefore, equivalent at best to a force of pm perfect warheads, while a force of m warheads of launch reliability one and terminal reliability p may be better.

Launch reliability is a function of booster reliability, guidance reliability against gross errors, and defense impact prediction capability, while terminal reliability is a function of guidance reliability given no gross error occurs, warhead detonation reliability, and defense impact prediction capability. The role played by defense impact prediction capability is an important one, since it allows the defender to shift some of the overall warhead unreliability from terminal unreliability (which forces the use of interceptors) to launch unreliability (which does not). Impact prediction capability therefore has a definite quantifiable value in terms of interceptors saved, and this appears a promising area for further study. One additional factor to be considered in assessing warhead reliability is the density of the target system. In a densely packed target system such as a missile field, warheads which fail to attack their assigned targets owing to guidance errors may in fact attack other nearby targets instead.

Interceptor launch reliability refers to the probability that an interceptor does not fail early enough for another

to be launched at the same warhead, and terminal reliability to the probability that it then destroys that warhead. In the former case, if the interceptor fails, another shot at the warhead is possible if there are more interceptors available; in the latter case the warhead penetrates to strike the target. A force of n interceptors of launch reliability p and terminal reliability one is thus at best equal to a force of pn perfect interceptors, and may actually be as good in some cases. A force of n interceptors of launch reliability one and terminal reliability p, on the other hand, is definitely inferior to a force of pn perfect interceptors.

We have not attempted a quantitative analysis of the battle with imperfect forces, but have included these qualitative observations in an attempt to provide some insight into the factors involved. The mobile defense game for warheads and interceptors both with launch reliability one and terminal reliabilities less than one has been solved by Matheson in [3].

Appendix

THE MOBILE DEFENSE GAME

The game may be described as follows: There are two players, the attacker and the defender. The defender has t targets and n interceptors with which to defend them. The attacker has m warheads with which to attack the targets. The defender assigns interceptors to targets without knowledge of the attacker's assignment and the attacker assigns warheads to targets without knowledge of the defender's assignment. Targets to which more warheads than interceptors are assigned are destroyed. The attacker wishes to maximize (and the defender to minimize) the number of targets destroyed.

Since the targets are of equal value, the payoff depends only on the number of targets destroyed, and not on which targets are actually destroyed. From this fact it is clear that once each player has decided how many targets should be assigned 0, 1, 2, ..., etc. units, the actual targets to which these units are assigned should be picked randomly from a uniform distribution over all possible target assignments. Each target will have the same a priori probability of destruction, and the expected number of targets destroyed will be the number of targets (t) multiplied by this probability of destruction. The actual number of units assigned by each side to each target will be a random variable whose distribution will depend on the strategy

chosen by that side. Under the optimal strategies for each side, the random variables corresponding to different targets will be identically distributed, though not independent.

An "approximating game" is first solved in which each player may choose probability distributions corresponding to desired assignment strategies, subject only to the constraint that these distributions have the proper means*. It is then shown that except in certain extreme cases these distributions are realizable through actual assignment strategies; hence except in these cases the solution of the game is given by the solution of the approximating game.

A.1. THE APPROXIMATING GAME

In the approximating game, a strategy for the attacker is a vector $x = (x_0, x_1, \ldots, x_m)$ such that $x_i \geq 0$ for $0 \leq i \leq m$, $\sum_{i=0}^{m} x_i = 1$ and $\sum_{i=0}^{m} ix_i = \mu$, where $\mu = m/t$ is the mean number of warheads per target. A strategy for the defender is a vector $y = (y_0, y_1, \ldots, y_n)$ such that $y_j \geq 0$ for $0 \leq j \leq n$, $\sum_{j=0}^{n} y_j = 1$ and $\sum_{j=0}^{n} jy_j = \nu$, where $\nu = n/t$ is the mean number of interceptors per target. We think of x_i as the probability with which i warheads are assigned to each target and y_j as the probability with which j interceptors are assigned to each target. It will not generally be possible to assign units to targets in the original game so as to realize every such distribution.

*It is this game which is solved in [2] and [4].

The payoff to the attacker (expected number of targets destroyed) in the approximating game is given by

$$M(x, y) = t \sum_{i=1}^{m} \sum_{j=0}^{i-1} x_i y_j = t \sum_{j=0}^{n} \sum_{i=j+1}^{m} x_i y_j .$$

That is, the expected number of targets destroyed is t times the probability that the number of warheads exceeds the number of interceptors.

To avoid the trivial cases when all targets may be destroyed or saved, it will be assumed henceforth that $\mu < n$, $\nu < m$, and $t \geq 2$.

Let us define

$$\ell_0 = \min\,(\mathrm{ip}[2\nu + \tfrac{t-1}{t}],\ m),$$

$$\ell_1 = \min\,(\mathrm{ip}[2\mu - 1],\ n),$$

$$\ell = \max\,(\ell_0,\ \ell_1),$$

where ip indicates the integer part. The number ℓ is the largest number of defending units the defender will ever wish to put at a single target. If $\ell = \ell_0$, the game is said to be <u>defense dominated</u>, and if $\ell = \ell_1$, the game is said to be <u>attack dominated</u>. (The cases $\ell = m$ or $\ell = n$ are special situations in which one side is so weak in absolute number of units that he cannot mount an attack or defense

as large as he would like at a single target simply because he does not have enough units.)

If the game is defense dominated ($\ell = \ell_0$), let

$$x_0^* = 1 - \frac{2\mu}{\ell + 1},$$

$$x_i^* = \frac{2\mu}{\ell(\ell + 1)} \qquad 1 \leq i \leq \ell, \tag{1}$$

$$x_i^* = 0 \qquad \ell < i \leq m,$$

and let

$$y_j^* = \frac{2(\ell - \nu)}{\ell(\ell + 1)} \qquad 0 \leq j < \ell,$$

$$y_\ell^* = 1 - \frac{2(\ell - \nu)}{\ell + 1}, \tag{2}$$

$$y_j^* = 0 \qquad \ell < j \leq n.$$

If the game is attack dominated ($\ell = \ell_1$), let

$$x_i^* = (1 - \frac{\mu}{\ell + 1})\frac{2}{\ell} \qquad 1 \leq i \leq \ell,$$

$$x_{\ell+1}^* = \frac{2\mu}{\ell + 1} - 1, \tag{3}$$

$$x_i^* = 0 \qquad i = 0 \text{ and } \ell + 1 < i \leq m,$$

and let

$$y_0^* = 1 - \frac{2\nu}{\ell + 1},$$

$$y_j^* = \frac{2\nu}{\ell(\ell + 1)} \qquad 1 \leq j \leq \ell, \qquad (4)$$

$$y_j^* = 0 \qquad \ell < j \leq n.$$

Theorem 1. x^* and y^* are optimal. The value of the approximating game is

$$v = \frac{2t\ \mu(\ell - \nu)}{\ell(\ell + 1)} \qquad \text{if } \ell = \ell_0$$

and

$$v = t\left(1 - \frac{2\ \nu(\ell + 1 - \mu)}{\ell(\ell + 1)}\right) \quad \text{if } \ell = \ell_1.$$

The calculations which show that x^* and y^* are strategies are omitted as routine.

Case 1. $\ell = \ell_0$. If $j \geq \ell$, then $\ell - j \leq 0$, hence for any y

$$M(x^*, y) = t\sum_{j=0}^{n} \sum_{i=j+1}^{m} x_i^* y_j$$

$$= \frac{2t\ \mu}{\ell(\ell + 1)} \sum_{j=0}^{\ell-1} y_j(\ell - j)$$

$$\geq \frac{2t\,\mu}{\ell(\ell + 1)} \sum_{j=0}^{n} y_j (\ell - j)$$

$$= \frac{2t\,\mu(\ell - \nu)}{\ell(\ell + 1)}$$

$$= v.$$

If $\ell < i \leq m$, then $\frac{2i}{\ell + 1} \left(\frac{\ell - \nu}{\ell}\right) \geq 1$, hence for any x,

$$M(x, y^*) = t \sum_{i=1}^{m} \sum_{j=0}^{i-1} x_i y_j^*$$

$$= t\left(\sum_{i=1}^{\ell} x_i \frac{2i(\ell - \nu)}{\ell(\ell + 1)} + \sum_{i=\ell+1}^{m} x_i\right)$$

$$\leq \frac{2t(\ell - \nu)}{\ell(\ell + 1)} \sum_{i=1}^{m} i\, x_i$$

$$= \frac{2t\,\mu(\ell - \nu)}{\ell(\ell + 1)}$$

$$= v.$$

Case 2. $\ell = \ell_1$. If $\ell + 1 < j \leq n$, then $1 - \frac{2j(\ell + 1 - \mu)}{\ell(\ell + 1)} \leq 0$, hence for any y,

$$M(x^*, y) = t \sum_{j=0}^{n} \sum_{i=j+1}^{m} x_i^* y_j$$

$$= t \sum_{j=0}^{\ell+1} y_j \left(1 - \frac{2j(\ell + 1 - \mu)}{\ell(\ell + 1)}\right)$$

$$\geq t \sum_{j=0}^{n} y_j \left(1 - \frac{2j(\ell + 1 - \mu)}{\ell(\ell + 1)}\right)$$

$$= t \left(1 - \frac{2\nu(\ell + 1 - \mu)}{\ell(\ell + 1)}\right)$$

$$= v.$$

If $\ell < i \leq m$, then $\ell + 1 - i \leq 0$, hence for any y

$$M(x, y^*) = t \sum_{i=1}^{m} \sum_{j=0}^{i-1} x_i y_j^*$$

$$= t\left(\sum_{i=1}^{\ell} x_i \left(1 - \frac{2\nu(\ell + 1 - i)}{\ell(\ell + 1)}\right) + \sum_{i=\ell+1}^{m} x_i\right)$$

$$\leq t \sum_{i=1}^{m} x_i \left(1 - \frac{2\nu(\ell + 1 + i)}{\ell(\ell + 1)}\right)$$

$$= t \left(1 - \frac{2\nu(\ell + 1 - \mu)}{\ell(\ell + 1)}\right)$$

$$= v.$$

A.2. ASSIGNMENT STRATEGIES

In the approximating game, strategies for the players are probability distributions on the nonnegative integers

subject to constraints on their means and ranges determined by the number of targets and forces available to the attacker and the defender. Actually, however, the attacker and defender are not free to choose any probability distribution on the nonnegative integers, but must choose from among those distributions which can be realized by an assignment of m warheads or n interceptors to t targets. The solutions given in the previous section are optimal only when they can actually be achieved. If, for example, $t = 3$, $n = 5$, and $m = 2$, the game is defense dominated with $\ell = 2$. The strategies given by equations (1) and (2) are $x^* = (5/9, 2/9, 2/9)$ and $y^* = (1/9, 1/9, 7/9, 0, 0, 0)$; the value of the approximating game is $M(x^*, y^*) = 2/9$. The only method of assigning 5 interceptors to 3 targets with no more than 2 interceptors per target, however, is to assign 2 interceptors to each of 2 targets and one interceptor to the remaining targets. The attacker's optimal strategy against this defense assignment is always to assign both of his warheads to a single target chosen at random; his expected payoff is then 1/3.

In the above example, ℓ was determined by the size of the attacking force rather than by the mean number of interceptors per target. This represents a special situation in which the maximum attack size is limited by the actual number of missiles available; this number is less than the optimal maximum attack size for the means μ and ν.

According to the next theorem, except in this situation and the corresponding situation when the maximum defense is limited by the total number of interceptors available, the optimal distributions x^* and y^* are obtainable.

Theorem 2. In the attack (defense) dominated game, the distribution y^* (x^*) is always attainable by the defender (attacker) and the distribution x^* (y^*) is attainable by the attacker (defender) unless $\ell = n$ ($\ell = m$).

The distributions x^* and y^* given by equations (1) – (4) are determined by the mean μ or ν, the integer ℓ, and the requirement that the distribution place equal probability on the integers 1 through ℓ (or 0 through $\ell - 1$) and the remaining probability on 0 (or on ℓ or on $\ell + 1$). Rather than attempting the more tedious task of calculating the actual values given by equations (1) – (4), we shall prove the theorem by describing methods of assigning warheads (or interceptors) to targets in such a manner that the number of warheads or interceptors assigned to a target picked at random will meet these requirements. (The choice of a target at random reflects the fact that the attacker (or defender), once he decides how many warheads (or interceptors) to assign to each of t targets, should make the actual assignment at random.) We describe, rather than explicitly construct, the assignment procedure because of the extreme notational difficulties which arise in a general construction. We

shall prove the theorem only for the attacker, since the distribution x^* in the defense dominated game is the same as y^* in the attack dominated game, while the distribution y^* in the defense dominated game can be obtained from x^* in the attack dominated game by considering the random variable $Y = \ell + 1 - X$.

The Attack Dominated Game

In describing the procedure to achieve x^* in the attack dominated game with $\ell = ip[2\mu - 1]$, we must consider several cases. The case $t = 3$ consists of three subcases, $m = (3\ell + 3)/2$, $m = (3\ell + 4)/2$, and $m = (3\ell + 5)/2$, which correspond to the three possible fractional parts $2\mu - 1$: 0, 1/3, and 2/3. The simplest of these is $m = (3\ell + 3)/2$. In this case, ℓ is odd. We choose integer j from a uniform distribution on $\{1, 2, \ldots, \ell\}$, and assign j, $(\ell - j + 2)/2$, and $(2\ell - j + 1)/2$ warheads to the three targets respectively if j is odd; or j, $(2\ell - j + 2)/2$, and $(\ell - j + 1)/2$ if j is even. Routine calculations show that this assignment procedure achieves the distribution x^*.

Next consider the case $m = (3\ell + 4)/2$. Using the procedure given above, we can assign $m + 1$ warheads to 3 targets with equal probability that each target receives $1, 2, \ldots, \ell + 1$ missiles, since $m + 1 = (3(\ell + 1) + 3)/2$. If the assignment resulting from the procedure is (a_1, a_2, a_3), with $a_1 + a_2 + a_3 = m + 1$, then use the assignment (a_1', a_2', a_3'), such that $a_i' = a_i - 1$ and

$a_j' = a_j$ for $i \neq j$, $i = 1, 2, 3$, with probability $(a_i - 1)/(m - 2)$. Since $\sum_{i=1}^{3} (a_i - 1) = m - 2$, and $\sum_{i=1}^{3} a_i' = m$, we have in fact described a procedure for assigning m warheads to 3 targets. Let X be the number of warheads assigned to a randomly chosen target. We must show that $P(X = 0) = 0$, $P(X = j) = c$ for some constant c and $1 \leq j \leq \ell$, and $P(X > \ell + 1) = 0$. Let Z be the number of warheads assigned by the first stage of the procedure (the assignment of m + 1 missiles). Then $P(Z = j) = 1/(\ell + 1)$ if $1 \leq j \leq \ell + 1$, $P(Z = j) = 0$ if $j = 0$ or $j > \ell + 1$, and $P(X = j|Z = j + 1) = j/(m - 2)$ and $P(X = j|Z = j) = (m - j - 1)/(m - 2)$ for $1 \leq j \leq \ell$. Therefore

$$P(X = 0) = P(Z = 0) + P(Z = 1)P(X = 0|Z = 1) = 0,$$

and for $1 \leq j \leq \ell$,

$$\begin{aligned} P(X = j) &= P(Z = j)P(X = j|Z = j) \\ &\quad + P(Z = j + 1)P(X = j|Z = j + 1) \\ &= \left(\frac{1}{\ell + 1}\right)\cdot\left(\frac{m - j + 1}{m - 2}\right) + \left(\frac{1}{\ell + 1}\right)\left(\frac{j}{m - 2}\right) \qquad (5) \\ &= \frac{(m - 1)}{(\ell + 1)(m - 2)}, \end{aligned}$$

which is what was to be shown.

If $m = (3\ell + 5)/2$, then $m - 1 = (3\ell + 3)/2$, and we can assign $m - 1$ of the warheads by the procedure outlined above for the case $m = (3\ell + 3)/2$. If the assignment resulting from this procedure for $m - 1$ warheads is (a_1, a_2, a_3), then we use the assignment (a_1', a_2', a_3') such that $a_1' = a_i + 1$ and $a_j' = a_j$ for $j \neq i$; $i = 1, 2, 3$, with probability $a_i/(m - 1)$. The proof that the resulting distribution of warheads at a randomly chosen target is x^* is similar to the proof in the case $m = (3\ell + 4)/2$, and is omitted.

The procedure for arbitrary t is as follows. Using the fact that $\ell = ip[2\mu - 1]$, we see that

$$m = \frac{t}{2}(\ell + 1) + b, \quad b < \frac{t}{2} . \tag{6}$$

Since m is an integer, if t is even b is also an integer. We divide the t targets into t/2 target pairs, and assign $\ell + 2$ warheads to each of b pairs and $\ell + 1$ warheads to each of the remaining $(t/2) - b$ pairs. Within each pair we assign j warheads to one target and the remaining warheads to the other target, where j is chosen from a uniform distribution on $\{1, 2, \ldots, \ell\}$ if $\ell + 1$ missiles are assigned to the target pair, and from a uniform distribution on $\{1, 2, \ldots, \ell + 1\}$ if $\ell + 2$ missiles are assigned. The routine calculations similar to those above show that this assignment procedure does achieve x^*.

If t is odd and greater than 3, we can rewrite equation (6) as

$$m = m_1 + m_2,$$

where $m_1 = (3\ell + 3)/2$ and $m_2 = [(t-3)(\ell + 1)]/2$ if ℓ is odd and $b = 0$; $m_1 = (3\ell + 5)/2$ and $m_2 = [(t-3)(\ell + 1)/2 + (b-1)]$ if ℓ is odd and $b > 0$; and $m_1 = (3\ell + 4)/2$ and $m_2 = [(t-3)(\ell + 1)/2 + (b - 1/2)]$ if ℓ is even. Since m is an integer, b is an integer if ℓ is odd and an odd multiple of 1/2 if ℓ is even. Therefore both m_1 and m_2 are integers, and the distribution x^* can be achieved by assigning m_1 warheads to 3 of the targets and m_2 warheads to the remaining $t - 3$ targets using the procedures outlined for $t = 3$ and t even above.

The Defense Dominated Game

To achieve x^* in the defense dominated game we must assign warheads to targets in such a way that the number of warheads assigned to a randomly chosen target is 1, 2, ..., or ℓ with equal probability and zero with the remaining probability necessary to achieve the proper mean. The choice of ℓ insures that $\ell \leq m \leq (\ell + 1)t/2$, so that both the range and the mean of the required distribution are compatible with the number of warheads and targets available.

If either t or $\ell + 1$ is even, and $m = (\ell + 1)t/2$ (the maximum number possible), the warheads can be assigned precisely as in the attack dominated game. If both t and $\ell + 1$ are odd, and $m = [(\ell + 1)t - 1]/2$, assignment can be made in a similar manner, although slight modifications are necessary to place the excess probability on zero rather than on $\ell + 1$ warheads. Consider 3 targets and $(3\ell + 2/2$ warheads. The targets may be thought of as each having $\ell + 1$ spaces to which warheads may be assigned, a total of $3\ell + 3$. When the warheads are assigned, $(3\ell + 4)/2$ empty spaces will remain; the number of empty spaces at a randomly chosen target should be 1, 2, ..., or ℓ with equal probability and $\ell + 1$ with the remaining probability. The empty spaces can therefore be assigned by the procedure given for warhead assignment in the attack dominated game, and the warheads assigned to the remaining spaces. The remaining $(\ell + 1)(t - 3)/2$ warheads can be assigned to the remaining $(t - 3)$ targets as before.

We have thus shown that the maximum number of warheads possible for a given value of ℓ can be assigned to achieve x^* in the defense dominated game. To complete the proof, we need only show that for $m \geq \ell$ the assignment procedure for m warheads can be derived from the assignment procedure for m + 1. To assign m warheads given an assignment procedure for m + 1, we simply assign m + 1 according to that procedure. We then select one of the targets to which warheads have been assigned, and remove one warhead,

selecting a target to which $j(1 \leq j \leq \ell)$ warheads have been assigned with probability $(\ell + 1 - j)/[H(\ell + 1) - m]$, where H is the number of targets to which at least one warhead has been assigned. Calculation of the resulting distribution of warheads is similar to that given in equation (5), but is complicated slightly by the fact that H is itself a random variable. As before, let Z be the number of warheads assigned to a randomly chosen target by the procedure for m + 1 warheads, and X the resulting number assigned after the removal of one warhead. Recall that we assume $P(Z = j) = p$ for some p and $1 \leq j \leq \ell$. Thus, for $1 \leq j < \ell$,

$$
\begin{aligned}
P(X=j) &= P(X=j|Z=j)P(Z=j) + P(X=j|Z=j+1)P(Z=j+1) \\
&= p[P(X=j|Z=j) + P(X=j|Z=j+1)] \\
&= p\sum_{h=1}^{t} P(H=h)[P(X=j|Z=j,H=h) + P(X=j|Z=j+1,H=h)] \\
&= p\sum_{h=1}^{t} [(1 - \frac{\ell+1-j}{h(\ell+1)-m}) + \frac{\ell+1-(j+1)}{h(\ell+1)-m})]P(H = h) \\
&= p\sum_{h=1}^{t} (\frac{h(\ell+1) - m-1}{h(\ell+1)-m})P(H = h) \\
&= c.
\end{aligned}
$$

A similar equation holds for $P(X = \ell)$. Note that the actual distribution of H was not needed for the calculation.

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10. ABSTRACT

A two-person zero-sum game is used to model the mobile defense problem in order to examine, in the context of terminal ballistic missile defense, the advantages that accrue to the defender from being able to deny the attacker knowledge of the defense deployment. The marginal exchange ratio is shown to be quite favorable to the defender using mobile interceptors if the fraction of the target system that he wishes to protect is small. A combination of mobile defense and deceptive basing appears to be an extremely attractive defense option, since the mobile defense raises the price the attacker must pay to destroy a single target, while the deceptive basing increases the number of targets at which he must pay that price.

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